

A proof of the Livingston conjecture

Avkhadiev F., Wirths K.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

Let D denote the open unit disc and $f : D \rightarrow \mathbb{C}$ be meromorphic and injective in D . We further assume that f has a simple pole at the point $p \in (0, 1)$ and an expansion $f(z) = \frac{1}{z-p} + a_0 + a_1(z-p) + \dots$. In particular, we consider functions f that map D onto a domain whose complement with respect to \mathbb{C} is convex. Because of the shape of $f(D)$ these functions will be called concave univalent functions with pole p and the family of these functions is denoted by $\text{Co}(p)$. It is proved that for fixed $p \in (0, 1)$ the domain of variability of the coefficient $a_n(f)$, $n \geq 2$, $f \in \text{Co}(p)$, is determined by the inequality $|a_n(f)| \leq n$. This settles two conjectures published by A. E. Livingston in 1994 and by Ch. Pommerenke and the authors of the present article in 2004. © Walter de Gruyter 2007.

<http://dx.doi.org/10.1515/FORUM.2007.007>
